CAN WE BORROW THE CONCEPT OF INDEPENDENT RELATION FROM LINEAR ALGEBRA IN SOME DISCRETE MATH APPLICATIONS

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Sperner family [1, 2], formally an antichain in the inclusion lattice over the power set of a universal set X, is also called an independent system. The independence is defined as the non-containing-ship between every pair of members. In other words, the dependence is defined as the existence of a containing-ship for some pairs. This is a relation between two members. In contrast, the dependence in linear algebra is defined by the relation between one member and one group (many members). We therefore ask if this relational difference for the Sperner family is appropriate, borrowing the concept from linear algebra? We study the above question by starting from investigating the purpose of independence definition arranged for the Sperner family.

If the purpose is to make the family compact, there should be no redundant member in the family. An independent system (the Sperner family) is thus equivalent to a family without any redundant member. A redundant member is clearly understood by words is a member, whose existence or not does not make any difference for the family. By this interpretation, the dependence relation is built between the redundant member and the rest of the family.

To check if there is a difference made by the suspicious redundant member, the originally вЂњstaticвЂќ member needs to be regarded as a function to have the ability to influence (make a difference). One simple arrangement is to regard the member as a Boolean function and the family (union of members) as a combination of functions. To be more specific, the member is a function composed of product (logic AND) of operands; the family is a function composed of a sum (logic OR) of products. For example, the member [01100] and the family [11000], [10100], [01100] are regarded as the Boolean functions bc and ab + ac + bc respectively.

To check if the family is compact (independent) needs to check for every member to be not redundant. The check for a specific member is to check if the reduced family (the original family excluding the specific member) behaves identically to the original family. The above two families (the reduced one the original one) are regarded as two Boolean functions. Two functions to be identical require the two corresponding outputs to be identical for every possible input. Therefore, we need to check the difference between the two outputs for every possible input. The requirements on checking every member (Boolean function) and every input (Boolean block) make the checking lengthy. To simplify and to visualize the checking, we design a full-pattern (the all possible Boolean block combination) image for the testing Boolean functions. In this sense, the checking work is implemented by an image processing. A full-pattern image for the case of the universal set X with cardinality five will be presented in the conference.

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References

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